LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **APRIL 2023**

MT 5505 – REAL ANALYSIS

Date: 29-04-2023 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

Part A

Answer ALL questions **(10 x 2 = 20)**

1. Define similar set, give an example.
2. Give examples of countable and uncountable sets.
3. Sketch the concept of open set.
4. Write the conditions for discrete metric space.
5. Give an example to show every continuous function need not be uniformly continuous.
6. Define complete metric space.
7. Define differentiability at a point.
8. Define local minimum, give an example.
9. What is meant by partition of a closed set $[a, b]$?
10. Define bounded variation.

Part B

Answer any FIVE questions **(5 x 8 = 40)**

11. State and prove Archimedean property.
12. Prove that every subset of a countable set is countable.
13. Let (X, d) be a metric space. Prove the following:
    (i) the union of an arbitrary collection of open sets in X is open in X.
    (ii) the intersection of an arbitrary collection of closed sets in X is closed in X.
14. Prove that every convergent sequence is a Cauchy sequence. Give an example to show the converse need not be true.
15. State and prove intermediate value theorem for continuity.
16. State and prove Rolle's theorem.
17. If f and g are both differentiable at a point $c \in X$, then prove that $f + g, fg$ and αf are differentiable at c, where α is a constant.
18. Define compact set and show that every compact subset of a metric space is complete.

Part C

Answer any TWO questions **(2 x 20 = 40)**

19. (a) State and prove Cauchy-Schwartz inequality.
    (b) Let $M = R^n$ and let $x = (x_1, x_2 \dots x_n), y = (y_1, y_2 \dots y_n)$ and $z = (z_1, z_2 \dots z_n) \in R^n$. Define $\mathrm{d}(\mathrm{x}, \mathrm{y}) = \{\sum_{k=1}^{n} (\mathrm{x_k} - \mathrm{y_k}^2\}^{\frac{1}{2}}$. Prove that (M, d) is a metric space. **(12+8)**
20. State and prove Taylor's Theorem.
21. (a) State and prove generalized mean value theorem.
    (b) Prove that the Euclidean Space R^k is complete. **(8+12)**
22. (a) State and prove uniform continuity theorem.
    (b) Let f be of bounded variation on $[a, b]$ and $c \in (a, b)$. Prove that f is of bounded variation on $[a, c]$ as well as on $[c, b]$ and $V_f[a, b] = V_f[a, c] + V_f[c, b]$. **(10+10)**

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